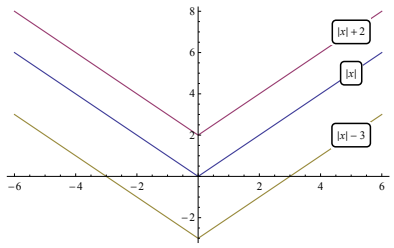
Business PreCalculus MATH 1643 Section 004, Spring 2014

Lesson 16: Transformations of Functions

In this lesson we learn hot to form a new function from an old one by performing certain operations. The graph of the new function is called the **transformation** of the graph of the old one. For example, the graph of $y = |x| + 1$ is a transformation of the graph of $y = |x|$.

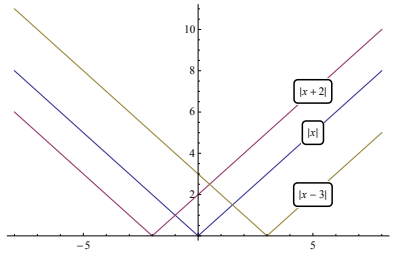
Definition 1. **Vertical Shifts:** *Let $d > 0$, the graph of* ***y=f(x)+d*** *is the graph of $y = f(x)$ shifted d units* ***up****. The graph of* ***y=f(x)-d*** *is the graph of $y = f(x)$ shifted d units* ***down****.*

Example 1. *The graph of $f(x) = |x| + 2$ is the graph of $f(x) = |x|$ shifted 2 units up, while the graph of $f(x) = |x| - 3$ is the graph of $f(x) = |x|$ shifted 3 units down.*



Definition 2. **Horizontal Shifts:** *Let $d > 0$, the graph of* ***y=f(x+d)*** *is the graph of $y = f(x)$ shifted d units* ***to the left****. The graph of* ***y=f(x-d)*** *is the graph of $y = f(x)$ shifted d units* ***to the right****.*

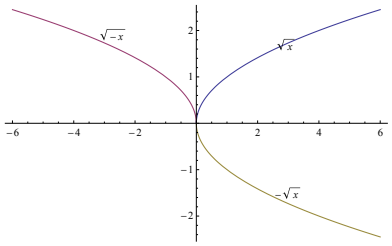
Example 2. *The graph of $f(x) = |x + 2|$ is the graph of $f(x) = |x|$ shifted 2 units to the left, while the graph of $f(x) = |x - 3|$ is the graph of $f(x) = |x|$ shifted 3 units to the right.*



Definition 3. **Reflection in the x-Axis:** *The graph of* ***y=-f(x)*** *is a* ***reflection*** *of the graph of $y = f(x)$ in the* ***x-axis****.*

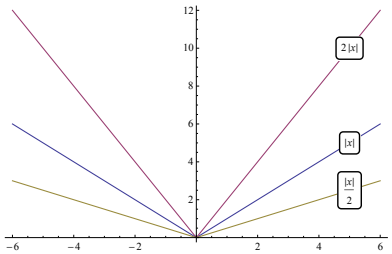
Definition 4. **Reflection in the y-Axis:** *The graph of* ***y=f(-x)*** *is a* ***reflection*** *of the graph of $y = f(x)$ in the* ***y-axis****.*

Example 3. *The graph of $f(x) = \sqrt{-x}$ is the reflection of the graph of $f(x) = \sqrt{x}$ on the y-axis, while the graph of $f(x) = -\sqrt{x}$ is the reflection of the graph of $f(x) = \sqrt{x}$ on the x-axis.*



Definition 5. **Vertical Stretching and Compressing:** *Let $a > 1$, then the graph of* ***y=af(x)*** *is a* ***vertical stretch*** *of the graph of $y = f(x)$ away from the* ***x-axis****. If $0 < a < 1$, then the graph of* ***y=af(x)*** *is a* ***vertical compression*** *of the graph of $y = f(x)$ toward the* ***x-axis****.*

Example 4. *The graph of $f(x) = 2|x|$ is the graph of $f(x) = |x|$ vertically stretched by multiplying each of its y-coordinates by 2. It is* ***twice as high*** *as the graph of $|x|$ at every real number x. The graph of $f(x) = \frac{1}{2}|x|$ is the graph of $f(x) = |x|$ vertically compressed by multiplying each of its y-coordinates by $\frac{1}{2}$. It is* ***half as high*** *as the graph of $|x|$ at every real number x.*



Definition 6. **Horizontal Stretching and Compressing:** *Let $0 < a < 1$, then the graph of $y = f(ax)$ is a* ***horizontal stretch*** *of the graph of $y = f(x)$ away from the* ***y-axis****. If $a > 1$, then the graph of $y = f(ax)$ is a* ***horizontal compression*** *of the graph of $y = f(x)$ toward the* ***y-axis****.*

Example 5. *The graph of $f(x) = (\frac{1}{2}x)^2$ is the graph of $f(x) = x^2$ horizontally stretched away from the y-axis. The graph of $f(x) = (2x)^2$ is the graph of $f(x) = x^2$ horizontally compressed toward the y-axis.*